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The tachyon potential in the sliver frame

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Abstract: We evaluate the tachyon potential in the Schnabl gauge through off-shell computations in the sliver frame. As an application of the results of our computations, we provide a strong evidence that Schnabl's analytic solution for tachyon condensation in open string field theory represents a saddle point configuration of the full tachyon potential. Additionally we verify that Schnabl's analytic solution lies on the minimum of the effective tachyon potential.

Keywords: Tachyon Condensation, String Field Theory

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Contents

1 Introduction

Schnabl's [1] description of an analytic solution for tachyon condensation has sparked renewed interest in string field theory in the last few years. The study of open string tachyon condensation on unstable branes in bosonic and superstring theory is interesting, since it involves three important conjectures made by Ashoke Sen [2, 3]. The first conjecture related the height of the tachyon potential at the true minimum to the tension of the D-brane; the second conjecture predicted existence of lump solutions with correct tensions, which describe lower dimensional D-branes; and the third conjecture stated that there are no physical excitations around the true minimum.[1] Witten's cubic or Chern-Simons open string field theory [9] has provided precise quantitative tests of these conjectures [10–24].

Since string field theory corresponds to second-quantized string theory, a point in its classical configuration space corresponds to a specific quantum state of the first quantized string theory. As shown in ref. [9], in order to describe a gauge invariant open string field theory we must include the full Hilbert space of states of the first quantized open string theory, including the b and c ghost fields. Witten's formulation of open string field theory is based on the following Chern-Simons action

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle\Phi, Q_B\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right], \tag{1.1}$$

where Q_B is the BRST operator of bosonic string theory, $*$ stands for Witten's star product, and the inner product $\langle\cdot,\cdot\rangle$ is the standard BPZ inner product. The string field Φ belongs to the full Hilbert space of the first quantized open string theory. The action has gauge invariance $\delta\Phi = Q_B\Lambda + \Phi * \Lambda - \Lambda * \Phi$.

[1] There are similar conjectures for the open string tachyon on a non-BPS D-brane and the tachyon living on the brane-antibrane pair [4–8].

The tachyon potential in Witten's cubic open string field theory has been computed numerically by an approximation scheme called level truncation [10–16]. This method is rather similar to the variational method in quantum mechanics and it was first used by Kostelecky and Samuel. The scheme is based on the realization that by truncating the string field Φ to its low lying modes (keeping only the Fock states with $L_0 < h$), one obtains an approximation that gets more accurate as the level h is increased. Therefore, the string field was traditionally expanded in the so-called Virasoro basis of L_0 eigenstates. However, it is well known that in this basis calculations involving the cubic interaction term becomes cumbersome and the three-string vertex that defines the star product in the string field algebra $\Phi_1 * \Phi_2$ is complicated [25–28]. We can overcome these technical issues (related to the definition of the star product) by using a new coordinate system [1].

The open string worldsheet is usually parameterized by a complex strip coordinate $w = \sigma + i\tau$, $\sigma \in [0, \pi]$, or by $z = -e^{-iw}$, which takes values on the upper half plane. As shown in [1], the gluing conditions entering into the geometrical definition of the star product simplify if one uses another coordinate system, $\tilde{z} = \arctan z$, in which the upper half plane looks as a semi-infinite cylinder of circumference π. In this new coordinate system, which we will henceforth call the *sliver frame*, it is possible to write down simple, closed expressions for arbitrary star products within the subalgebra generated by Fock space states. Elements of this subalgebra are known in the literature [29–31] as wedge states with insertions.

The simplicity of the definition of the star product in the sliver frame allows us to solve analytically the string field equation of motion [1]

$$Q_B\Phi + \Phi * \Phi = 0\,. \tag{1.2}$$

Schnabl's analytic solution $\Phi \equiv \Psi$ was obtained by expanding the string field Ψ in a basis of $\mathcal{L}_0$ eigenstates, where $\mathcal{L}_0$ is the zero mode of the worldsheet energy momentum tensor $T_{\tilde{z}\tilde{z}}$ in the $\tilde{z}$ coordinate. By a conformal transformation it can be written as

$$\mathcal{L}_0 = \oint \frac{dz}{2\pi i}(1+z^2)\arctan z\, T_{zz}(z) = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k}\,, \tag{1.3}$$

where the L_n's are the ordinary Virasoro generators with zero central charge $c = 0$ of the total matter and ghost conformal field theory. The coefficients of the $\mathcal{L}_0$ level expansion of the string field Ψ are obtained by plugging Ψ into the equation of motion. Remarkably, imposing the Schnabl gauge condition $\mathcal{B}_0\Psi = 0$[2] and truncating the equation of motion (but not the string field) to the subset of states up to some maximal $\mathcal{L}_0$ eigenvalue lead to a system of algebraic equations for the coefficients, which can be solved exactly level by level. It was shown that the analytic solution Ψ reproduces the desired value for the normalized vacuum energy predicted from Sen's first conjecture [32–37]

$$2\pi^2\left[\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right] = -1\,. \tag{1.4}$$

[2] $\mathcal{B}_0$ is the zero mode of the b ghost in the $\tilde{z}$ coordinate, which can be defined by a conformal transformation in a similar manner as $\mathcal{L}_0$.

To date, it is still an open question how to construct an explicit gauge transformation to prove the equivalence between Schnabl's analytic solution and the numerical solution found in the level truncation scheme in the Siegel gauge. These two solutions are believed to be the same tachyon vacuum solution. Evidence which supports this statement is given by evaluating gauge invariants quantities on these solutions, namely, the vacuum energy and the gauge invariant overlap [38–42]. The computation of these gauge invariant quantities does not tell us much about the type of configuration associated to the solutions. Since, in general, a solution to the equation of motion corresponds to extremal configurations, at first sight we do not know that the solution will correspond to a minimum, maximum or saddle point configuration of the theory. In the case of string field theory a direct way to find the kind of configuration associated with the solution is to compute the off-shell tachyon potential. In this paper, we compute the tachyon potential in the so-called sliver frame. By extremizing this potential, we search for extremal configurations, and remarkably it turns out that Schnabl's analytic solution for tachyon condensation represents a saddle point configuration of the full tachyon potential.

Regarding to the effective tachyon potential, let us point out that this potential is non-unique. In general we can compute the effective tachyon potential as follows. By decomposing the string field as $\Phi = t\mathcal{T} + \chi$, where $\mathcal{T}$ is the tachyonic part of the string field (the zero momentum tachyon state), while χ is an arbitrary string field which belongs to the gauge fixed Hilbert space linearly independent of the first term $\mathcal{T}$. To obtain the effective tachyon potential, we must integrate out the string field χ, this is done by inserting the string field Φ into the action, solving the equation of motion for χ and plugging back to the action. The resulting expression, as a function of the single variable t is the effective potential. Therefore we see that the effective potential computed in this way is non-unique since it depends on the choice of an specific gauge to fix the string field Φ and the choice of $\mathcal{T}$. For instance, traditionally $\mathcal{T}$ is taken to be $c_1|0\rangle$ and the gauge used is the Siegel gauge $b_0\Phi = 0$.

Usually when we compute the effective tachyon potential in the Siegel gauge [14–16], the fields which are integrated out correspond to the perturbative Fock space of states with mass greater than the tachyon mass. In the case of the Schnabl gauge, to find the effective tachyon potential in the sliver frame, instead of integrating out fields in the state space used in the Siegel gauge, we integrate out fields with $\mathcal{L}_0$ eigenvalue greater than -1, which corresponds to the $\mathcal{L}_0$ eigenvalue of the tachyon state $\tilde{c}_1|0\rangle$. This means that the effective tachyon potential we compute in the sliver frame is different from the old effective tachyon potential computed in the Siegel gauge. As an application of the results of our computations, we address the question of whether Schnabl's analytic solution corresponds to a minimum configuration of the effective tachyon potential, and we find that this is indeed the case.

We have chosen $\mathcal{T} = \tilde{c}_1|0\rangle$ as the tachyonic part of the string field since we consider this choice to be the most natural one from the perspective of Schnabl's coordinates (the sliver frame). Choosing insertions on other wedge states does not seem to be natural, except for insertions over the sliver or the identity $\tilde{c}_1|\infty\rangle$, $\tilde{c}_1|\mathcal{I}\rangle$, nevertheless both of these options are singular [43–46]. We believe that any choice other than $\mathcal{T} = \tilde{c}_1|0\rangle$ would require some motivation. One of the main motivations for computing the tachyon potential is to investigate its branch structure, for example, to see if the perturbative and non-perturbative vacua are connected on the same branch of the potential, to see if there are other branches with other solutions, and to understand the off-shell limitations of the gauge-choice. If we choose $\mathcal{T}$ to be the tachyon vacuum solution, the resulting potential is less interesting in this respect. From a technical point of view our choice $\mathcal{T} = \tilde{c}_1|0\rangle$ appears to be the less involved one.

Another remark we would like to comment is related to the choice of basis for integrating out the remaining string field χ. The effective tachyon potential should not depend on the choice of basis even if the basis are related by a somewhat singular transformation like the transformation between the L_0 with the $\mathcal{L}_0$ basis. This statement should be true provided that we can manage the singularity by a suitable regularization prescription. For instance, the use of Padé resummation techniques would eventually be needed [34, 36].

This paper is organized as follows. In section 2, we introduce Witten's formulation of open bosonic string field theory. After writing down the form of the cubic action, we define the two- and three-string interaction vertex. To evaluate these interaction vertices, we use CFT correlators defined on the sliver frame. In section 3, we study the structure of the tachyon potential in some detail using the $\mathcal{L}_0$ level expansion of the string field in the Schnabl gauge. By extremizing the potential, we provide a strong evidence that Schnabl's analytic solution corresponds to a saddle point configuration of the theory. A summary and further directions of exploration are given in section 4.

2 Open bosonic string field theory revisited

In this section, we are going to review briefly some aspects of Witten's cubic open string field theory which will be relevant to the purposes of this paper.

2.1 Witten's string field theory

Witten's formulation of open string field theory is axiomatic. The space of string fields $\mathcal{H}$ is taken to be an associative noncommutative algebra provided with a $\mathbf{Z}_2$ grading and a $*$-multiplication operation on $\mathcal{H}$. The multiplication law $*$ satisfies the property that the $\mathbf{Z}_2$ degree of the product $a * a'$ of two elements a, $a' \in \mathcal{H}$ is $(-1)^a(-1)^{a'}$, where $(-1)^a$ is the $\mathbf{Z}_2$ degree of a. There exists an odd derivation Q acting on $\mathcal{H}$ as $Q(a * a') = Q(a) * a' + (-1)^a a * Q(a')$. Q is also required to be nilpotent: $Q^2 = 0$. These properties remind us of the BRST operator Q_B. The final ingredient is the integration, which maps $a \in \mathcal{H}$ to a complex number $\int a \in \mathbb{C}$. This operation is linear, $\int(a + a') = \int a + \int a'$, and satisfies $\int(a * a') = (-1)^{aa'} \int(a' * a)$ where $(-1)^{aa'}$ is defined to be -1 only if both a and a' are odd elements of $\mathcal{H}$. Also, $\int Q(a) = 0$ for any a.

Let us take a close look at the $*$-multiplication. As discussed in detail in [9], for the multiplication to be associative, i.e. $(a * a') * a'' = a * (a' * a'')$, we must interpret the $*$-operation as gluing two half-strings together. Take two strings S, S', whose excitations are described by the string fields a and a', respectively. Each string is labeled by a coordinate $0 \leq \sigma \leq \pi$ with the midpoint $\sigma = \pi/2$. Then the gluing procedure is as follows: The right hand piece $\pi/2 \leq \sigma \leq \pi$ of the string S and the left hand piece $0 \leq \sigma \leq \pi/2$ of the string S' are glued together, and what is left is the string-like object consisting of the left half of S and the right half of S'. This is the product $S * S'$ in the gluing prescription, and the resulting string state on $S * S'$ corresponds the string field $a * a'$. Since $a * a'$ and $a' * a$ are in general thought of as representing completely different elements, their agreement under the integration $\int (a * a') = (-1)^{aa'} \int (a' * a)$ (up to a sing $(-1)^{aa'}$) suggests that the integration procedure still glues the remaining sides of S and S'. If we restate it for a single string $S * S'$, the left hand piece is sewn to the right hand piece under the integration.

Using the above definition of the $*$-multiplication and the integration $\int$, we can write the string field theory action as follows

$$\mathcal{S} = -\frac{1}{g^2} \int \left(\frac{1}{2} \Phi * Q_B \Phi + \frac{1}{3} \Phi * \Phi * \Phi \right), \tag{2.1}$$

where g is the open string coupling constant, Φ is the string field which belongs to the full Hilbert space of the first quantized open string theory. The algebra is equipped with a $\mathbf{Z}_2$ grading given by the ghost number, if we define $\#_{\rm gh}$ as an operator that counts the ghost number of its argument, then we have that: $\#_{\rm gh}(\Phi) = 1\,,\ \#_{\rm gh}(Q_B) = 1\,,\ \#_{\rm gh}(*) = 0\,,\ \#_{\rm gh}(b) = -1\,,\ \#_{\rm gh}(c) = 1$. The action (2.1) is invariant under the infinitesimal gauge transformation $\delta\Phi = Q_B \Lambda + \Phi * \Lambda - \Lambda * \Phi$, where Λ is a gauge parameter with $\#_{\rm gh}(\Lambda) = 0$. In the conformal field theory (CFT) prescription, the action (2.1) is evaluated as the two- and three-point correlation function.

Since the action (2.1) has been derived quite formally, it is not suitable for concrete calculations. In particular, $*$ and $\int$ operations have been defined only geometrically as the gluing procedure. In the next subsection we will argue the methods of computation based on conformal field theory techniques.

2.2 The two- and three-string vertex

The open string worldsheet is parameterized by a complex strip coordinate $w = \sigma + i\tau$, $\sigma \in [0, \pi]$ or by $z = -e^{-iw}$ which takes values in the upper half plane (UHP). As shown in ref. [1], the gluing conditions entering into the geometrical definition of the star product simplify if one uses another coordinate system, $\tilde{z} = \arctan z$, in which the upper half plane looks as a semi-infinite cylinder C_π of circumference π, we have called this new coordinate system as the sliver frame.

For purposes of computations, the sliver frame seems to be the most natural one since the conformal field theory in this new coordinate system remains easy. As in the case of the upper half plane, we can define general n-point correlation functions on C_π which can be readily found in terms of correlation functions defined on the upper half plane by a conformal mapping,

$$\langle \phi_1(\tilde{x}_1) \cdots \phi_n(\tilde{x}_n) \rangle_{C_\pi} = \langle \tilde{\phi}_1(\tilde{x}_1) \cdots \tilde{\phi}_n(\tilde{x}_n) \rangle_{\rm UHP}\,, \tag{2.2}$$

where the fields $\tilde{\phi}_i(\tilde{x}_i)$ are defined as conformal transformation $\tilde{\phi}_i(\tilde{x}_i) = \tan \circ \phi_i(\tilde{x}_i)$. In general $f \circ \phi$ denotes a conformal transformation of a field ϕ under a map f, for instance if ϕ represents a primary field of dimension h, then $f \circ \phi$ is defined as $f \circ \phi(x) = (f'(x))^h \phi(f(x))$.

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The two-string vertex which appears in the string field theory action is the familiar BPZ inner product of conformal field theory.[3] It is defined as a map $\mathcal{H} \otimes \mathcal{H} \to \mathbb{R}$

$$\langle \phi_1, \phi_2 \rangle = \langle \mathcal{I} \circ \phi_1(0) \phi_2(0) \rangle_{\mathrm{UHP}} \,, \tag{2.3}$$

where $\mathcal{I} : z \to -1/z$ is the inversion symmetry. For states defined on the sliver frame $|\tilde{\phi}_i >$ the two-string vertex can be written as

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle = \langle \mathcal{I} \circ \tilde{\phi}_1(0) \tilde{\phi}_2(0) \rangle_{\mathrm{UHP}} = \left\langle \phi_1 \left(\frac{\pi}{2} \right) \phi_2(0) \right\rangle_{C_\pi} . \tag{2.4}$$

As we can see in this last expression, we evaluate the two-string vertex at two different points, namely at $\pi/2$ and 0 on C_π. This must be the case since the inversion symmetry maps the point at $z = 0$ on the upper half plane to the point at infinity, but the point at infinity is mapped to the point $\pm\pi/2$ on C_π.

The three-string vertex is a map $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \to \mathbb{R}$, and it is defined as a correlator on a surface formed by gluing together three strips representing three open strings. For states defined on the sliver frame $|\tilde{\phi}_i >$ the three-string vertex can be written as

$$\langle \tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3 \rangle = \left\langle \phi_1 \left(\frac{3\pi}{4} \right) \phi_2 \left(\frac{\pi}{4} \right) \phi_3 \left(-\frac{\pi}{4} \right) \right\rangle_{C_{\frac{3\pi}{2}}} . \tag{2.5}$$

Here the correlator is taken on a semi-infinite cylinder $C_{\frac{3\pi}{2}}$ of circumference $3\pi/2$. Also, this correlator can be evaluated on the semi-infinite cylinder C_π of circumference π. We only need to perform a simple conformal map (scaling) $s : \tilde{z} \to \frac{2}{3}\tilde{z}$ which brings the region $C_{\frac{3\pi}{2}}$ to C_π, and the correlator is given by

$$\langle \tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3 \rangle = \left\langle s \circ \phi_1 \left(\frac{3\pi}{4} \right) s \circ \phi_2 \left(\frac{\pi}{4} \right) s \circ \phi_3 \left(-\frac{\pi}{4} \right) \right\rangle_{C_\pi} . \tag{2.6}$$

Note that the scaling transformation s is implemented by $U_3 = (2/3)^{\mathcal{L}_0}$, where $\mathcal{L}_0$ is the zero mode of the worldsheet energy momentum tensor $T_{\tilde{z}\tilde{z}}(\tilde{z})$ in the $\tilde{z}$ coordinate,

$$\mathcal{L}_0 = \oint \frac{d\tilde{z}}{2\pi i} \tilde{z} T_{\tilde{z}\tilde{z}}(\tilde{z}) \,. \tag{2.7}$$

By a conformal transformation it can be expressed as

$$\mathcal{L}_0 = \oint \frac{dz}{2\pi i} (1 + z^2) \arctan z T_{zz}(z) = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2 - 1} L_{2k} \,, \tag{2.8}$$

where the L_n's are the ordinary Virasoro generators (with zero central charge) of the full (matter plus ghost) conformal field theory.

[3] Recall that the BPZ conjugate for the modes of an holomorphic field ϕ of dimension h is given by $\mathrm{bpz}(\phi_n) = (-1)^{n+h} \phi_{-n}$.

2.3 Correlation functions

In this subsection we list correlation functions evaluated on the semi-infinite cylinder C_π. As already mentioned, the relation between correlation functions evaluated on the upper half plane and those evaluated on the semi-infinite cylinder is given by conformal transformation.

Employing the definition of the conformal transformation $\tilde{c}(x) = \cos^2(x)c(\tan x)$ of the c ghost and its anticommutation relations with the operators Q_B, $\mathcal{B}_0$ and B_1,[4]

$$\{Q_B, \tilde{c}(z)\} = \tilde{c}(z)\partial\tilde{c}(z)\,, \tag{2.9}$$
$$\{\mathcal{B}_0, \tilde{c}(z)\} = z\,, \tag{2.10}$$
$$\{B_1, \tilde{c}(z)\} = 1\,, \tag{2.11}$$

we obtain the following basic correlation functions,

$$\langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle = \sin(x-y)\sin(x-z)\sin(y-z)\,, \tag{2.12}$$
$$\langle\tilde{c}(x)Q_B\tilde{c}(y)\rangle = -\sin(x-y)^2\,, \tag{2.13}$$
$$\langle\tilde{c}(x)\mathcal{B}_0\tilde{c}(y)\tilde{c}(z)\tilde{c}(w)\rangle = y\langle\tilde{c}(x)\tilde{c}(z)\tilde{c}(w)\rangle - z\langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle + w\langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle\,, \tag{2.14}$$
$$\langle\tilde{c}(x)\tilde{c}(y)\mathcal{B}_0\tilde{c}(z)\tilde{c}(w)\rangle = z\langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle - w\langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle\,, \tag{2.15}$$
$$\langle\tilde{c}(x)B_1\tilde{c}(y)\tilde{c}(z)\tilde{c}(w)\rangle = \langle\tilde{c}(x)\tilde{c}(z)\tilde{c}(w)\rangle - \langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle + \langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle\,, \tag{2.16}$$
$$\langle\tilde{c}(x)\tilde{c}(y)B_1\tilde{c}(z)\tilde{c}(w)\rangle = \langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle - \langle\tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle\,. \tag{2.17}$$

To compute correlation functions involved in the evaluation of the string field theory action, the following contour integrals will be very useful,

$$\sigma(a) \equiv \oint \frac{dz}{2\pi i} z^a \sin(2z) = \frac{\theta(-a-2)}{\Gamma(-a)}((-1)^a+1)(-1)^{\frac{2-a}{2}}2^{-a-2}\,, \tag{2.18}$$
$$\varsigma(a) \equiv \oint \frac{dz}{2\pi i} z^a \cos(2z) = \frac{\theta(-a-1)}{\Gamma(-a)}((-1)^a-1)(-1)^{\frac{1-a}{2}}2^{-a-2}\,, \tag{2.19}$$

$$\begin{aligned}
\mathcal{F}(a_1,a_2,a_3,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3) &\equiv \oint \frac{dx_1dx_2dx_3}{(2\pi i)^3} x_1^{a_1}x_2^{a_2}x_3^{a_3}\langle\tilde{c}(\alpha_1x_1+\beta_1)\tilde{c}(\alpha_2x_2+\beta_2)\tilde{c}(\alpha_3x_3+\beta_3)\rangle \\
&= \frac{1}{\alpha_1^{a_1+1}\alpha_2^{a_2+1}\alpha_3^{a_3+1}}\Bigg[\\
&\delta_{a_3,-1}\frac{\big(\sigma(a_1)\sigma(a_2)+\varsigma(a_1)\varsigma(a_2)\big)\sin(2(\beta_1-\beta_2))+\big(\sigma(a_1)\varsigma(a_2)-\varsigma(a_1)\sigma(a_2)\big)\cos(2(\beta_1-\beta_2))}{4} \\
&+\delta_{a_2,-1}\frac{\big(\varsigma(a_1)\sigma(a_3)-\sigma(a_1)\varsigma(a_3)\big)\cos(2(\beta_1-\beta_3))-\big(\varsigma(a_1)\varsigma(a_3)+\sigma(a_1)\sigma(a_3)\big)\sin(2(\beta_1-\beta_3))}{4} \\
&+\delta_{a_1,-1}\frac{\big(\varsigma(a_2)\varsigma(a_3)+\sigma(a_2)\sigma(a_3)\big)\sin(2(\beta_2-\beta_3))+\big(\sigma(a_2)\varsigma(a_3)-\varsigma(a_2)\sigma(a_3)\big)\cos(2(\beta_2-\beta_3))}{4}\Bigg],
\end{aligned} \tag{2.20}$$

[4]The operators $\mathcal{B}_0$ and $B_1 \equiv \mathcal{B}_{-1}$ are modes of the b ghost which are defined on the semi-infinite cylinder coordinate as $\mathcal{B}_n = \oint \frac{dz}{2\pi i}(1+z^2)(\arctan z)^{n+1}b(z)$.

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where $\theta(n)$ is the unit step (Heaviside) function which is defined as

$$\theta(n) = \begin{cases} 0, & \text{if } n < 0 \\ 1, & \text{if } n \geq 0\,. \end{cases} \tag{2.21}$$

Let us list a few non-trivial correlation functions which involve operators frequently used in the $\mathcal{L}_0$ basis, namely, $\hat{\mathcal{L}}^n$ ($\hat{\mathcal{L}} \equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger$), $\hat{\mathcal{B}}$ ($\hat{\mathcal{B}} \equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger$), $U_r = \left(\frac{2}{r}\right)^{\mathcal{L}_0}$ and the $\tilde{c}(z)$ ghost

$$\begin{aligned} &\langle \mathrm{bpz}(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1} U_r^\dagger U_r \tilde{c}(x) \tilde{c}(y) \rangle = \\ &= \oint \frac{dz_1 dx_1}{(2\pi i)^2} \frac{(-2)^{n_1} n_1!\, x_1^{p_1-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2} \left\langle \tilde{c}\left(x_1+\frac{\pi}{2}\right) \tilde{c}\left(\frac{4}{z_1 r}x\right) \tilde{c}\left(\frac{4}{z_1 r}y\right) \right\rangle, \end{aligned} \tag{2.22}$$

$$\begin{aligned} &\langle \mathrm{bpz}(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1} \hat{\mathcal{B}} U_r^\dagger U_r \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle = \\ &= -\delta_{p_1,0} \oint \frac{dz_1}{2\pi i} \frac{(-2)^{n_1} n_1!}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2} \left\langle \tilde{c}\left(\frac{4}{z_1 r}x\right) \tilde{c}\left(\frac{4}{z_1 r}y\right) \tilde{c}\left(\frac{4}{z_1 r}z\right) \right\rangle \\ &\quad + \oint \frac{dz_1 dx_1}{(2\pi i)^2} \frac{(-2)^{n_1} n_1!\, x_1^{p_1-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2} \times \\ &\qquad \times \left\langle \tilde{c}\left(x_1+\frac{\pi}{2}\right) \mathcal{B}_0 \tilde{c}\left(\frac{4}{z_1 r}x\right) \tilde{c}\left(\frac{4}{z_1 r}y\right) \tilde{c}\left(\frac{4}{z_1 r}z\right) \right\rangle, \end{aligned} \tag{2.23}$$

$$\begin{aligned} &\langle \mathrm{bpz}(\tilde{c}_{p_1}) \mathrm{bpz}(\tilde{c}_{p_2}) \hat{\mathcal{L}}^{n_1} \hat{\mathcal{B}} U_r^\dagger U_r \tilde{c}(x) \tilde{c}(y) \rangle = \\ &= -\delta_{p_2,0} \oint \frac{dz_1 dx_1}{(2\pi i)^2} \frac{(-2)^{n_1} n_1!\, x_1^{p_1-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1} \times \\ &\qquad \times \left\langle \tilde{c}\left(x_1+\frac{\pi}{2}\right) \tilde{c}\left(\frac{4}{z_1 r}x\right) \tilde{c}\left(\frac{4}{z_1 r}y\right) \right\rangle \\ &\quad + \delta_{p_1,0} \oint \frac{dz_1 dx_2}{(2\pi i)^2} \frac{(-2)^{n_1} n_1!\, x_2^{p_2-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1} \times \\ &\qquad \times \left\langle \tilde{c}\left(x_2+\frac{\pi}{2}\right) \tilde{c}\left(\frac{4}{z_1 r}x\right) \tilde{c}\left(\frac{4}{z_1 r}y\right) \right\rangle \\ &\quad + \oint \frac{dz_1 dx_1 dx_2}{(2\pi i)^3} \frac{(-2)^{n_1} n_1!\, x_1^{p_1-2} x_2^{p_2-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1} \times \\ &\qquad \times \left\langle \tilde{c}\left(x_1+\frac{\pi}{2}\right) \tilde{c}\left(x_2+\frac{\pi}{2}\right) \mathcal{B}_0 \tilde{c}\left(\frac{4}{z_1 r}x\right) \tilde{c}\left(\frac{4}{z_1 r}y\right) \right\rangle, \end{aligned} \tag{2.24}$$

where the "*bpz*" acting on the modes of the $\tilde{c}(z)$ ghost stands for the usual BPZ conjugation which in the $\mathcal{L}_0$ basis is defined as follows

$$\mathrm{bpz}(\tilde{\phi}_n) = \oint \frac{dz}{2\pi i} z^{n+h-1} \tilde{\phi}\left(z+\frac{\pi}{2}\right), \tag{2.25}$$

for any primary field $\tilde{\phi}(z)$ with weight h. The action of the BPZ conjugation on the modes of $\tilde{\phi}(z)$ satisfies the following useful property

$$U_r^{\dagger -1} \mathrm{bpz}(\tilde{\phi}_n) U_r^\dagger = \left(\frac{2}{r}\right)^{-n} \mathrm{bpz}(\tilde{\phi}_n)\,. \tag{2.26}$$

Correlation functions which involve only modes of the $\tilde{c}(z)$ ghost can be expressed in terms of the contour integral (2.20) as follows

$$\begin{aligned}
\langle \tilde{c}_p \tilde{c}_q \tilde{c}_r \rangle &= \oint \frac{dxdydz}{(2\pi i)^3} x^{p-2} y^{q-2} z^{r-2} \langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle \\
&= \mathcal{F}(p-2, q-2, r-2, 1, 0, 1, 0, 1, 0)\,, && (2.27)\\
\langle \mathrm{bpz}(\tilde{c}_p)\tilde{c}_q \tilde{c}_r \rangle &= \oint \frac{dxdydz}{(2\pi i)^3} x^{p-2} y^{q-2} z^{r-2} \Big\langle \tilde{c}\left(x+\frac{\pi}{2}\right)\tilde{c}(y)\tilde{c}(z)\Big\rangle \\
&= \mathcal{F}\left(p-2, q-2, r-2, 1, \frac{\pi}{2}, 1, 0, 1, 0\right)\,, && (2.28)\\
\langle \mathrm{bpz}(\tilde{c}_p)\mathrm{bpz}(\tilde{c}_q)\tilde{c}_r \rangle &= \oint \frac{dxdydz}{(2\pi i)^3} x^{p-2} y^{q-2} z^{r-2} \Big\langle \tilde{c}\left(x+\frac{\pi}{2}\right)\tilde{c}\left(y+\frac{\pi}{2}\right)\tilde{c}(z)\Big\rangle \\
&= \mathcal{F}\left(p-2, q-2, r-2, 1, \frac{\pi}{2}, 1, \frac{\pi}{2}, 1, 0\right)\,. && (2.29)
\end{aligned}$$

To evaluate correlators involving modes of the $\tilde{c}(z)$ ghost and insertions of operators $\hat{\mathcal{L}}^n$, $\hat{\mathcal{B}}$, we can use the basic correlators (2.14)–(2.17) and the definition of $\hat{\mathcal{L}}^n \equiv (-2)^n n! \oint \frac{dz}{2\pi i}\frac{1}{(z-2)^{n+1}} U_z^\dagger U_z$. For instance, as a pedagogical illustration let us compute a correlator involving a $\hat{\mathcal{L}}^n$ insertion,

$$\begin{aligned}
\langle \mathrm{bpz}(\tilde{c}_p)(\mathcal{L}_0+\mathcal{L}_0^\dagger)^n \tilde{c}_q\tilde{c}_r\rangle &= (-2)^n n! \oint \frac{dz_1}{2\pi i}\frac{1}{(z_1-2)^{n+1}} \langle \mathrm{bpz}(\tilde{c}_p) U_{z_1}^\dagger U_{z_1} \tilde{c}_q\tilde{c}_r\rangle && (2.30)\\
&= (-2)^n n! \oint \frac{dz_1}{2\pi i}\frac{(\frac{2}{z_1})^{-p-q-r}}{(z_1-2)^{n+1}} \langle \mathrm{bpz}(\tilde{c}_p)\tilde{c}_q\tilde{c}_r\rangle \\
&= (-1)^n n! \binom{p+q+r}{n} \mathcal{F}\left(p-2, q-2, r-2, 1, \frac{\pi}{2}, 1, 0, 1, 0\right)\,,
\end{aligned}$$

where we have used the following useful contour integral $\oint \frac{dz}{2\pi i}\frac{z^m}{(z-a)^{n+1}} = \binom{m}{n} a^{m-n}$.

Correlators involving the $*$-product can be computed using the results of this subsection. For instance, let us compute the correlator $\langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_2}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_3}|0\rangle$,

$$\begin{aligned}
&\langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_2}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_3}|0\rangle = \\
&\quad= \frac{(-2)^{n_2+n_3} n_2! n_3!}{(2\pi i)^4} \oint \frac{dz_2 dz_3 dx_2 dx_3\, x_2^{p_2-2} x_3^{p_3-2}}{(z_2-2)^{n_2+1}(z_3-2)^{n_3+1}} \times \\
&\qquad \times \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, U_{z_2}^\dagger U_{z_2}\tilde{c}(x_2)|0\rangle * U_{z_3}^\dagger U_{z_3}\tilde{c}(x_3)|0\rangle \\
&\quad= \frac{(-2)^{n_2+n_3} n_2! n_3!}{(2\pi i)^4} \oint \frac{dz_2 dz_3 dx_2 dx_3\, x_2^{p_2-2} x_3^{p_3-2}}{(z_2-2)^{n_2+1}(z_3-2)^{n_3+1}} \times \\
&\qquad \times \Big\langle \mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1} U_r^\dagger U_r \tilde{c}\left(x_2+\frac{\pi}{4}(z_3-1)\right)\tilde{c}\left(x_3-\frac{\pi}{4}(z_2-1)\right)\Big\rangle \\
&\quad= \frac{(-1)^{n_1+n_2+n_3} 2^{2n_1+n_2+n_3-2p_1-4} n_1! n_2! n_3!}{(2\pi i)^3} \oint \frac{dz_1 dz_2 dz_3\, z_1^{p_1+2} r^{p_1+2-n_1}}{(z_1-2)^{n_1+1}(z_2-2)^{n_2+1}(z_3-2)^{n_3+1}} \times \\
&\qquad \times \mathcal{F}\left(p_1-2, p_2-2, p_3-2, 1, \frac{\pi}{2}, \frac{4}{z_1 r}, \frac{\pi(z_3-1)}{z_1 r}, \frac{4}{z_1 r}, \frac{\pi(1-z_2)}{z_1 r}\right)\,, \qquad (2.31)
\end{aligned}$$

where we have defined $r \equiv z_2 + z_3 - 1$.

3 The tachyon potential

To compute the effective tachyon potential in a particular gauge, it is necessary to specify which fields are being integrated out. Usually, when we compute the effective tachyon potential in the Siegel gauge, the fields which are integrated out correspond to the perturbative Fock space of states with mass greater than the tachyon mass [14, 15].

In this section, in order to find the effective tachyon potential in the sliver frame, instead of integrating out fields in the state space mentioned in the previous paragraph, we are going to integrate out fields with $\mathcal{L}_0$ eigenvalue greater than the $\mathcal{L}_0$ eigenvalue of the tachyon state $\tilde{c}_1|0\rangle$. This means that the effective tachyon potential we compute is different from the old effective tachyon potential computed in the Siegel gauge. As we already commented in the introduction, we choose the state $\tilde{c}_1|0\rangle$ as the tachyonic state since it is the most natural one from the perspective of Schnabl's coordinates (the sliver frame). Choosing insertions on other wedge states does not seem to be natural, except for insertions over the sliver or the identity $\tilde{c}_1|\infty\rangle$, $\tilde{c}_1|\mathcal{I}\rangle$, nevertheless both of these options are singular [43–46].

3.1 The effective tachyon potential in the Schnabl gauge

As in the case of the Siegel gauge, in the Schnabl gauge we could perform an analysis of the tachyon potential by performing computations in the $\mathcal{L}_0$ level truncation. We are going to define the level of a state as the eigenvalue of the operator $N = \mathcal{L}_0 + 1$. This definition is adjusted so that the zero momentum tachyon $\tilde{c}_1|0\rangle$ is at level zero.

Having defined the level number of states contained in the level expansion of the string field, level of each term in the action is also defined to be the sum of the levels of the fields involved. For instance, if states $\tilde{\phi}_1$, $\tilde{\phi}_2$, $\tilde{\phi}_3$ have level n_1, n_2, n_3 respectively, we assign level $n_1 + n_2 + n_3$ to the interaction term $\langle\tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3\rangle$. When we say level (m, n), we mean that the string field includes all terms with level $\leq m$ while the action includes all terms with level $\leq n$.

In this paper, we want to study questions related to the appearance of a stable vacuum in the theory when the tachyon and other scalar fields acquire nonzero expectation values. Because all the questions we will address involve Lorentz-invariant phenomena, we can restrict attention to scalar fields in the string field expansion. We write the string field expansion in terms of scalar fields as

$$\Psi = \sum_{i=0}^{\infty} x_i|\psi^i\rangle\,, \tag{3.1}$$

where in the $\mathcal{L}_0$ level expansion the state $|\psi^i\rangle$ is built by applying the modes of the $\tilde{c}(z)$ ghost and the operators $(\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n$, $\mathcal{B}_0 + \mathcal{B}_0^\dagger$ on the $\mathrm{SL}(2,\mathbb{R})$ invariant vacuum $|0\rangle$. The first term in the expansion is given by the zero-momentum tachyon $|\psi^0\rangle = \tilde{c}_1|0\rangle$. We will restrict our attention to an even-twist and ghost-number one string field Ψ satisfying the Schnabl gauge $\mathcal{B}_0\Psi = 0$. Choosing a particular gauge prevents the inclusion of 'almost' flat directions which would have correspond to gauge degrees of freedom in the potential ('almost' is in quotation marks, since level truncation destroys gauge symmetry). The tachyon potential we want to evaluate is defined as

$$V = 2\pi^2 \left[\frac{1}{2}\langle \Psi, Q_B \Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle \right]. \tag{3.2}$$

The effective tachyon potential can be determined by starting with the complete set of terms in the potential truncated at some level (m, n), fixing a value for x_0, solving for all coefficients x_i, $i \geq 1$, and plugging them back into the potential to rewrite it as a function of x_0.

In order to explain the procedure for finding the effective tachyon potential, let us first set all components of the string field Ψ to zero except for the first coefficient x_0. This state will be said to be of level zero. Thus, we take

$$\Psi = x_0 \tilde{c}_1 |0\rangle \,. \tag{3.3}$$

Plugging (3.3) into the definition (3.2), we get the zeroth approximation to the tachyon potential,

$$V^{(0,0)} = 2\pi^2 \left[-\frac{x_0^2}{2} + \frac{27\sqrt{3}x_0^3}{64} \right]. \tag{3.4}$$

To compute corrections to this result, we need to include higher level fields in our analysis. The analysis can be simplified by noting that the potential (3.2) has a twist symmetry under which all coefficients of odd-twist states change sign, whereas coefficients of even-twist states remain unchanged. Therefore coefficients of odd-twist states at levels above $\tilde{c}_1|0\rangle$ must always appear in the action in pairs, and they trivially satisfy the equations of motion if set to zero. Thus, we look for Ψ containing only even-twist states.

Taking into account the considerations above, at the next level we find that the string field is given by

$$\Psi = x_0 \tilde{c}_1|0\rangle - 2x_1(\mathcal{L}_0 + \mathcal{L}_0^\dagger)\tilde{c}_1|0\rangle - 2x_1(\mathcal{B}_0 + \mathcal{B}_0^\dagger)\tilde{c}_0\tilde{c}_1|0\rangle \,, \tag{3.5}$$

where the coefficients of the expansion were chosen so that Ψ satisfies the Schnabl gauge, $\mathcal{B}_0\Psi = 0$. Substituting this level expansion of the string field (3.5) into (3.2) we get the (1,3) level approximation to the potential

$$V^{(1,3)} = 2\pi^2 \left[-\frac{x_0^2}{2} + \frac{27\sqrt{3}x_0^3}{64} + \left(\frac{27}{8}\sqrt{3} - \frac{9}{8}\pi\right) x_0^2 x_1 + \left(\frac{9}{2}\sqrt{3} - 3\pi + \frac{2\pi^2}{\sqrt{3}}\right) x_0 x_1^2 \right]. \tag{3.6}$$

Since the effective tachyon potential depends on the single variable x_0 which corresponds to the tachyon coefficient, we are going to integrate out the variable x_1. Using the partial derivative of the potential, $\partial_{x_1} V^{(1,3)} = 0$, we can write the variable x_1 in terms of x_0

$$x_1 = \frac{27\pi - 81\sqrt{3}}{216\sqrt{3} - 144\pi + 32\sqrt{3}\pi^2}\, x_0 \,. \tag{3.7}$$

By plugging back (3.7) into the potential (3.6) to rewrite it as a function of the single variable x_0, we obtain the effective potential

$$V_{\text{eff}}^{(1,3)} = 2\pi^2 \left[-\frac{x_0^2}{2} + \frac{486\sqrt{3}\pi - 2187 + 405\pi^2}{3456\sqrt{3} - 2304\pi + 512\sqrt{3}\pi^2} x_0^3 \right] . \tag{3.8}$$

Extending our analysis to the next level, we are going to use the string field Ψ satisfying the Schnabl gauge $\mathcal{B}_0\Psi = 0$ expanded up to level two states,

$$\begin{aligned} \Psi \,=\, & x_0\tilde{c}_1|0\rangle - 2x_1(\mathcal{L}_0 + \mathcal{L}_0^\dagger)\tilde{c}_1|0\rangle - 2x_1(\mathcal{B}_0 + \mathcal{B}_0^\dagger)\tilde{c}_0\tilde{c}_1|0\rangle + x_2\tilde{c}_{-1}|0\rangle \\ & - x_3(\mathcal{L}_0 + \mathcal{L}_0^\dagger)^2\tilde{c}_1|0\rangle - 2x_3(\mathcal{L}_0 + \mathcal{L}_0^\dagger)(\mathcal{B}_0 + \mathcal{B}_0^\dagger)\tilde{c}_0\tilde{c}_1|0\rangle \,. \end{aligned} \tag{3.9}$$

To obtain the level (2,6) potential, we plug the string field (3.9) into the definition (3.2). By using some correlation functions derived in the previous section, we arrive to the following potential

$$\begin{aligned} V^{(2,6)} = 2\pi^2 \Bigg[& -\frac{x_0^2}{2} + \frac{27\sqrt{3}x_0^3}{64} + \left(\frac{27}{8}\sqrt{3} - \frac{9}{8}\pi\right)x_0^2x_1 + \left(\frac{9}{2}\sqrt{3} - 3\pi + \frac{2\pi^2}{\sqrt{3}}\right)x_0x_1^2 \\ & + x_0x_2 - \frac{3}{4}\sqrt{3}x_0^2x_2 - \frac{16\pi^2x_1^2x_2}{9\sqrt{3}} - x_2^2 + \frac{x_0x_2^2}{\sqrt{3}} + \left(\frac{8\pi}{27} - \frac{8}{3\sqrt{3}}\right)x_1x_2^2 - 2x_0x_3 \\ & + \left(\frac{3\pi}{4} - \frac{9}{8}\sqrt{3} + \frac{5\pi^2}{8\sqrt{3}}\right)x_0^2x_3 - \frac{8\pi^4x_1^2x_3}{81\sqrt{3}} - \frac{7\pi^2x_0x_2x_3}{9\sqrt{3}} + \left(\frac{8}{243}\pi^3 - \frac{8\pi^2}{3\sqrt{3}}\right)x_1x_2x_3 \\ & + \left(\frac{16\pi^2}{81\sqrt{3}} - \frac{8}{3\sqrt{3}} + \frac{16\pi}{27}\right)x_2^2x_3 + \left(\frac{2\pi^5}{2187} - \frac{98\pi^4}{243\sqrt{3}}\right)x_1x_3^2 + \frac{17\pi^4x_0x_3^2}{324\sqrt{3}} \\ & + \left(\frac{16\pi^3}{243} - \frac{8\pi^2}{3\sqrt{3}} - \frac{16\pi^4}{243\sqrt{3}}\right)x_2x_3^2 + \left(\frac{4\pi^5}{2187} - \frac{98\pi^4}{243\sqrt{3}} - \frac{28\pi^6}{6561\sqrt{3}}\right)x_3^3 \Bigg] . \end{aligned} \tag{3.10}$$

As we can see, starting at level (2,6), coefficients other than the tachyon coefficient x_0 are no longer quadratic, therefore we cannot exactly integrate out all these non-tachyonic coefficients (x_i, $i \geq 1$). Therefore, we are forced to use numerical methods to study the effective tachyon potential. We have used Newton's method to find the zeros of the partial derivatives of the potential. For a fixed value of the tachyon coefficient x_0, there are in general many solutions of the equations for the remaining coefficients x_i, $i \geq 1$, which correspond to different branches of the effective potential. We are interested in the branch connecting the perturbative with the nonperturbative vacuum and having a minimum value which agrees with the one predicted from Sen's first conjecture.

Applying the numerical approach described above, we have integrated out the variables x_1, x_2 and x_3 appearing in the potential (3.10). At this level, we found that the shape of the effective tachyon potential which connects the perturbative with the nonperturbative vacuum is given by the graph shown in figure 1. For reference we have plotted the effective tachyon potential up to level (3,9). The minimum value of the level (2,6) effective potential $V_{\text{eff}}^{(2,6)}$ occurs at $x_{0,min} = 0.7023612173$, and its depth gets the value of -1.0466220796 which is 4.66% greater than the conjectured value (1.4). At this level, we have noted that our algorithm becomes unstable for values of the tachyon coefficient between $0 < x_0 < 0.47$,

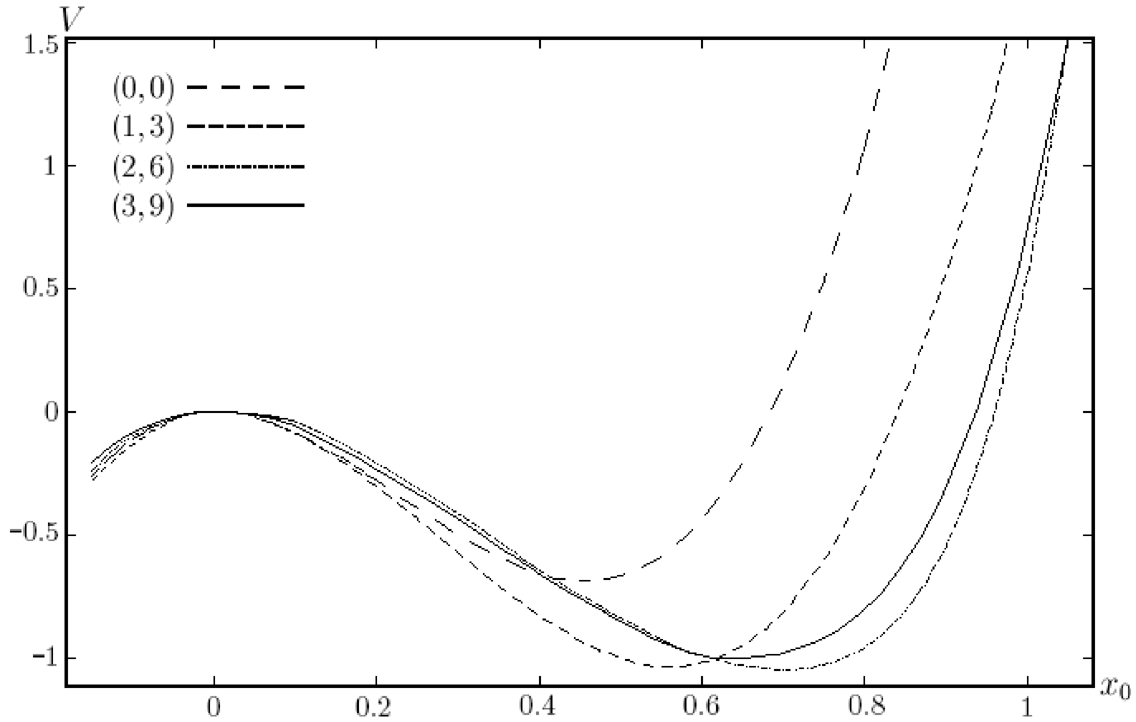

Figure 1. Effective tachyon potential at different levels.

this may indicate that the branch which contain the perturbative with the nonperturbative vacuum meets one or more other branches which play the role of attractors. In fact, we have found that there is a new branch which meets the physical branch[5] at $x_0 \approx 0.47$. This new branch contains the extremal points $x_0 = 0.2205432494$, $x_1 = -0.0150554001$, $x_2 = -0.2576803891$, $x_3 = 0.1491283766$, which are solutions to the equations coming from the partial derivatives of the potential (3.10). We have excluded this new branch (generated by these points) since it does not contain the nonperturbative vacuum.

We could continue to perform higher level computations, since these computations follow the same procedures shown above, but at this point we only want to comment about the results. Higher level computations reveal that the effective tachyon potential has the profile found in the lower level cases. However, at higher levels the algorithm used to compute the effective tachyon potential fails to converge outside the region $-0.014 < x_0 < 0.701$. This result indicates that the effective tachyon potential has branch points near $x_0 \approx -0.014$ and $x_0 \approx 0.701$, where the nontrivial vacuum appears at $x_0 = 0.636$. The locations of these branch points appear to converge under $\mathcal{L}_0$ level truncation to fixed values.

While the perturbative and nonperturbative vacua both lie on the effective potential curve between these branch points, the existence of these branch points prompts us to ask what cubic string field theory can say about the effective tachyon potential beyond these branch points. In particular, an issue of some interest is how the effective tachyon potential

[5] We refer to the physical branch as the branch which connects the perturbative with the nonperturbative vacuum.

behaves for large negative values of x_0. A possible physical reason why our algorithm fails to converge outside the region $-0.014 < x_0 < 0.701$ might be analogous to the case of the effective potential found in the usual L_0 level expansion, where the the existence of these branch points is related to the validity of the Siegel gauge. It is then possible that in the $\mathcal{L}_0$ level expansion the singularities previously found in the effective tachyon potential are gauge artifacts arising from the boundary of the region of validity of Schnabl gauge. We leave the analysis of this possiblity for further research.

Up to the level that we have explored with our computations, it is worth remarking that the depth of the effective potential is converging to the conjectured value (1.4). For instance, at level (5,15) the minimum value of the effective potential occurs at $x_{0,min} = 0.6368018630$, and its depth takes on the value -0.9993346627 which is 99.93% of the conjectured value.

Another remark is that at the minimum of the effective tachyon potential, the tachyon coefficient x_0 is approaching the analytical value of $2/\pi$, which interestingly is the same value of the tachyon coefficient in Schnabl's analytical solution when expanded in the $\mathcal{L}_0$ basis. In the next subsection we will give additional comments on this important observation.

To conclude this subsection, let us note that if we go to high enough level, the depth of the effective tachyon potential should diverge since the minimum of the effective potential should correspond to the value of the action evaluated on Schnabl's solution truncated at some finite $\mathcal{L}_0$ level. In order to regularize the value of the minimum of the effective potential, the use of Padé resummation techniques would eventually be needed [36].

3.2 The stable vacuum and Schnabl's solution

In this subsection we want to address the connection between the configuration found by extremizing the tachyon potential and Schnabl's analytic solution.

In order to compare the analytic solution with the one obtained by the methods shown in the previous subsection, let us write Schnabl's analytic solution up to level-two states

$$\begin{aligned}\Psi_{\text{analytic}} = {} & \frac{2}{\pi}\tilde{c}_1|0\rangle + \frac{1}{2\pi}(\mathcal{L}_0+\mathcal{L}_0^\dagger)\tilde{c}_1|0\rangle + \frac{1}{2\pi}(\mathcal{B}_0+\mathcal{B}_0^\dagger)\tilde{c}_0\tilde{c}_1|0\rangle + \frac{\pi}{48}\tilde{c}_{-1}|0\rangle \\ & + \frac{1}{24\pi}(\mathcal{L}_0+\mathcal{L}_0^\dagger)^2\tilde{c}_1|0\rangle + \frac{1}{12\pi}(\mathcal{L}_0+\mathcal{L}_0^\dagger)(\mathcal{B}_0+\mathcal{B}_0^\dagger)\tilde{c}_0\tilde{c}_1|0\rangle \,. \end{aligned} \tag{3.11}$$

This solution was found by solving the string field equation of motion [1]. In general, a solution to the equation of motion corresponds to extremal configurations. We are not guaranteed that the solution will correspond to a minimum, maximum or saddle point configuration of the tachyon potential. Certainly, as we have seen in the previous subsection, the solution lies on the minimum configuration of the effective tachyon potential. Next we are going to check whether Schnabl's solution is a saddle point configuration of the full tachyon potential.[6]

By using Fermat's theorem, the potential extremums of a multivariable function $f(x_0, \cdots, x_N)$, with partial derivative $\partial_i f \equiv \frac{\partial}{\partial x_i} f(x_0, \cdots, x_N)$, $i = 0, \cdots, N$ are found

[6]By the full tachyon potential we mean the tachyon potential without integrating out the coefficients (x_i, $i \geq 1$).

Level (2,6)	Level (3,9)	Level (4,12)	Level (5,15)	Coeff. Schnabl's solution
0.702361217	0.629070893	0.632254043	0.636801863	0.636619772
0.165917159	0.163408594	0.169249348	0.160355988	0.159154943
0.165917159	0.163408594	0.169249348	0.160355988	0.159154943
0.036787327	0.093717432	0.102813347	0.064453673	0.065449846
0.044922378	0.005498242	0.002281681	0.011760238	0.013262911
0.089844757	0.010996484	0.004563363	0.023520476	0.026525823

Table 1. The six first coefficients of the string field level expansion corresponding to the saddle point configuration of the full tachyon potential computed up to level (5,15). The last column shows the corresponding analytical value for these coefficients, taken from Schnabl's solution expanded in the $\mathcal{L}_0$ basis (3.11).

by solving an equation in $\partial_i f = 0$. Fermat's theorem gives only a necessary condition for extreme function values, and some stationary points are saddle points (not a maximum or minimum). A test that can be applied at a critical point $x \equiv (x_0, \cdots, x_N)$ is by using the Hessian matrix, which is defined as $H_{ij} \equiv \partial_i \partial_j f$. If the Hessian is positive definite at x, then f attains a local minimum at x. If the Hessian is negative definite at x, then f attains a local maximum at x. If the Hessian has both positive and negative eigenvalues then x is a saddle point for f.

We claim that Schnabl's analytic solution corresponds to a saddle point configuration of the full tachyon potential. Evidence supporting our claim is found by computing the string field corresponding to the extremal points[7] obtained from extremizing the full tachyon potential, and by computing the respective eigenvalues of the Hessian matrix. We have performed this analysis up to level (5,15). The results are shown in tables 1 and 2. In the first table 1 we have compared the first six coefficients of the analytical $\mathcal{L}_0$ level expansion of the solution (3.11) with those obtained from extremizing the full tachyon potential. In the second table 2, we show the respective eigenvalues of the Hessian matrix. It seems that some eigenvalues of the Hessian matrix does not have pattern of convergence when the level is increasing. We should attribute the origin of this divergence to the fact that Schnabl's analytic solution when expanded in the new bases of $\mathcal{L}_0$ eigenstates results in an asymptotic expansion [37]. This issue is in analogy with the problem of computing the depth of the effective tachyon potential at higher levels. As we already pointed out, the depth of the effective potential should diverge since the minimum of the effective potential should correspond to the value of the string field action evaluated on Schnabl's solution truncated at some finite $\mathcal{L}_0$ level.

4 Summary and discussion

We have given in detail a prescription for computing the tachyon potential in the sliver frame. As we have seen, calculations are performed more easily in this frame than in the

[7]Let us emphasize that these extremal points are the points corresponding to the minimum configuration of the effective tachyon potential.

Levels (m,n)	Eigenvalues of the Hessian matrix
(2,6)	327.495421, 223.737479, -35.453340, 11.654594
(3,9)	-1695.539743, 918.029302, 368.749341, -327.871269 97.186964, -16.995283, 13.192470
(4,12)	-26154.959292, -13583.490319, 5971.278019, 503.332849 -384.826805, 295.055986, -64.304441, 62.708283 16.044610, -14.620229, 6.552447
(5,15)	-1.445596×10^{6}, -333771.902128, 226359.468852, 14180.406985 4335.817536, -2974.472506, 774.905979, -680.652950, 5.811130 445.004921, 255.059728, -142.316817, 56.987331 -16.017131, -11.715162, -5.517987, 2.755357

Table 2. Eigenvalues of the Hessian matrix corresponding to the extremal points of the full tachyon potential at different levels.

usual Virasoro basis of L_0 eigenstates. For instance, in the old basis the evaluation of the cubic interaction term using CFT methods requires cumbersome computations of finite conformal transformations for non-primary fields. The simplicity of the definition of the $*$-product in the new basis allows us to overcome these difficulties.

Since one aim of this paper was to answer the question whether Schnabl's analytic solution corresponds to a saddle point configuration of the full tachyon potential, we have focused our attention to a string field satisfying the Schnabl gauge. Since the computation of the tachyon potential does not require us to choose a specific gauge condition, we can use another gauge for the string field. It would be interesting to find connections between those family of solutions computed in different gauges which gives the right value for the vacuum energy. For instance, in recent work [37] a new simple solution for the tachyon condensation was analyzed and an explicit gauge transformation which connects the new solution to the original Schnabl's solution was constructed [1].

We have provided a strong evidence that Schnabl's analytic solution corresponds to a saddle point configuration of the full tachyon potential, and furthermore we have shown that the solution lies on the minimum of the effective tachyon potential. Nevertheless, there remain two important issues regarding the vacuum solution. The first is related to the computation of the analytic solution in the Siegel gauge. The second is to construct an explicit gauge transformation which connects Schnabl's solution to the one found using the Siegel gauge.

An issue that could be addressed using the methods outlined in this work would be the computation of the effective tachyon potential in the sliver frame for the case of the cubic superstring field theory. The profile of the effective potential in this theory is very puzzling since the tachyon has vanishing expectation value at the local minimum of the effective potential, so the tachyon vacuum sits directly below the perturbative vacuum [47].

Finally, while we have not developed the details here, our methods should be applicable to computations in Berkovits's superstring field theory. The relevant string field theory is non-polynomial [48], but since the theory is based on Witten's associative star product, the methods discussed in this paper would apply with minor modifications. It would certainly be desirable to test the brane-antibrane annihilation conjecture analytically [49].

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